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ERROR ANALYSES USING DIFFERENCED ONE-WAY DOPPLER MEASUREMENTS FROM TDRSS

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ABSTRACT

This technical memorandum documents a study performed under Task Assignment 584, Tracker Calibration, to evaluate errors incurred when a user spacecraft is tracked using differenced one-way Doppler measurements via the Tracking and Data Relay Satellite System (TDRSS). The document describes the modeling used and the results obtained for a variety of circular user orbits. An error analysis of the bilateration tracking of the TDRSS relay satellites using ground transponders is also described.

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SECTION 1 - INTRODUCTION

The Tracking and Data Relay Satellite System (TDRSS) will permit tracking data to be processed in configurations not available from ground tracking or from satellite-to-satellite tracking (SST) with a single relay satellite. One of the experimental TDRSS methods involves the use of the Doppler difference measurement, which is computed as the difference between simultaneous SST one-way Doppler measurements taken via the two operational relay satellites. Although this mode of processing will be possible only during the intervals when the target spacecraft is visible from both relay satellites, it has the advantage of minimizing the effects of errors in the assumed frequency of the oscillator aboard the target spacecraft.

Since some of the information present in the two individual Doppler measurements is lost when only the difference is used, estimates are required of the accuracy with which target satellite orbits of various types can be determined using the Doppler difference data only. This document reports the results of a study undertaken to provide Doppler difference error analyses for a number of orbital configurations and tracking schedules.

SECTION 2 - METHODS USED IN THE ANALYSIS

2.1 SOFTWARE

The error analysis runs were performed using a slightly streamlined version of the method described in Section 4.3 of Reference 1. The Navigation Analysis Program (NAP) was used to simulate the Doppler data separately for each relay satellite, generating internal files of measurements and partial derivatives. These files were then processed by a standalone Doppler Difference (DOPDIF) Program to produce a similarly formatted file of Doppler difference measurements and partial derivatives; this file was read by NAP to generate the normal matrix of solve-for and consider parameters. The normal matrix was then analyzed and propagated by the NAP Covariance Analyzer (NAPCOV). In addition, standard NAP and NAPCOV runs were used to analyze the bilateration tracking of the TDRSS relay satellites using ground transponders.

2.2 DOPPLER DIFFERENCE MEASUREMENT AND PARTIAL DERIVATIVES

This subsection summarizes the computations performed by the program DOPDIF to generate the Doppler difference measurement and partial derivatives; also given in the subsection are the approximate magnitudes of some of the quantities involved in the computations.

The Doppler difference measurement, D, is computed by subtracting the one-way SST Doppler measurement via TDRS West, D_W, from the corresponding one-way Doppler measurement via TDRS East, D_E, i.e.,

$$D = D_E - D_W$$

The units for these measurements are in hertz, and their sign is positive for target orbits of inclination less than 90 degrees, variable for polar or near-polar orbits, and negative for orbits of inclination greater than 90 degrees. (See Tables 3-5 through 3-14 given in Section 3 of this document for some typical values obtained for the Doppler difference measurement.) Although not used in the error analysis, the Doppler difference residuals are also computed as the difference between the individual one-way Doppler residuals,

$$\Delta D = \Delta D_E - \Delta D_W$$

Partial derivatives with respect to the target state vector, drag coefficient, station coordinates, and geopotential coefficients are also computed as differences between the corresponding single-relay partial derivatives, i.e.,

$$\frac{\partial D}{\partial x} = \frac{\partial D_E}{\partial x} - \frac{\partial D_W}{\partial x}$$

Partial derivatives with respect to the relay state vectors and solar pressure coefficients are the individual Doppler partial derivatives with the appropriate sign as follows:

$$\frac{\partial D}{\partial x_E} = \frac{\partial D_E}{\partial x_E}$$

$$\frac{\partial D}{\partial x_W} = -\frac{\partial D_W}{\partial x_W}$$

Partial derivatives with respect to a bias in the target oscillator frequency, f_0, are computed using the fact that the Doppler difference measurement is always proportional to f_0, i.e.,

$$D = kf_0$$

where k is a function of the satellite positions and velocities. Therefore,

$$\frac{\partial D}{\partial f_0} = k = \frac{D}{f_0}$$

The partial derivative is the Doppler difference measurement value divided by the S-Band oscillator frequencey, f_0, which was assumed to be 2000 megahertz.

The bias produced in a Doppler difference measurement of 80 kilohertz by an oscillator bias, Δf_0, of 1000 hertz can be computed as

$$\Delta D = \frac{\partial D}{\partial f_0} \Delta f_0 = \frac{D}{f_0} \Delta f_0 = \frac{80,000}{2 \times 10^9} \times 1000 \text{ hertz} = 0.04 \text{ hertz}$$

This compares favorably with the bias of 1000 hertz produced in the individual one-way Doppler measurements.

A rate change $\dot{f}_0$ in the oscillator frequency, f_0, affects the Doppler difference measurement as follows. The one-way signal path lengths from the target to the relay to the ground are taken as R_E via the eastern relay and as R_W via the western relay. Their signals received simultaneously on the ground at time T will have been transmitted by the target oscillator at time $T - R_E/c$ and $T = R_W/c$, respectively, where c is the speed of light. Since the signal via TDRS East was transmitted at time $(R_W - R_E)/c$ after the one via TDRS West, the oscillator frequency via TDRS East will be greater than that via TDRS West by the quantity

$$\frac{R_W - R_E}{c} \dot{f}_0$$

which will produce a bias of

$$\frac{R_W - R_E}{c} \dot{f}_0$$

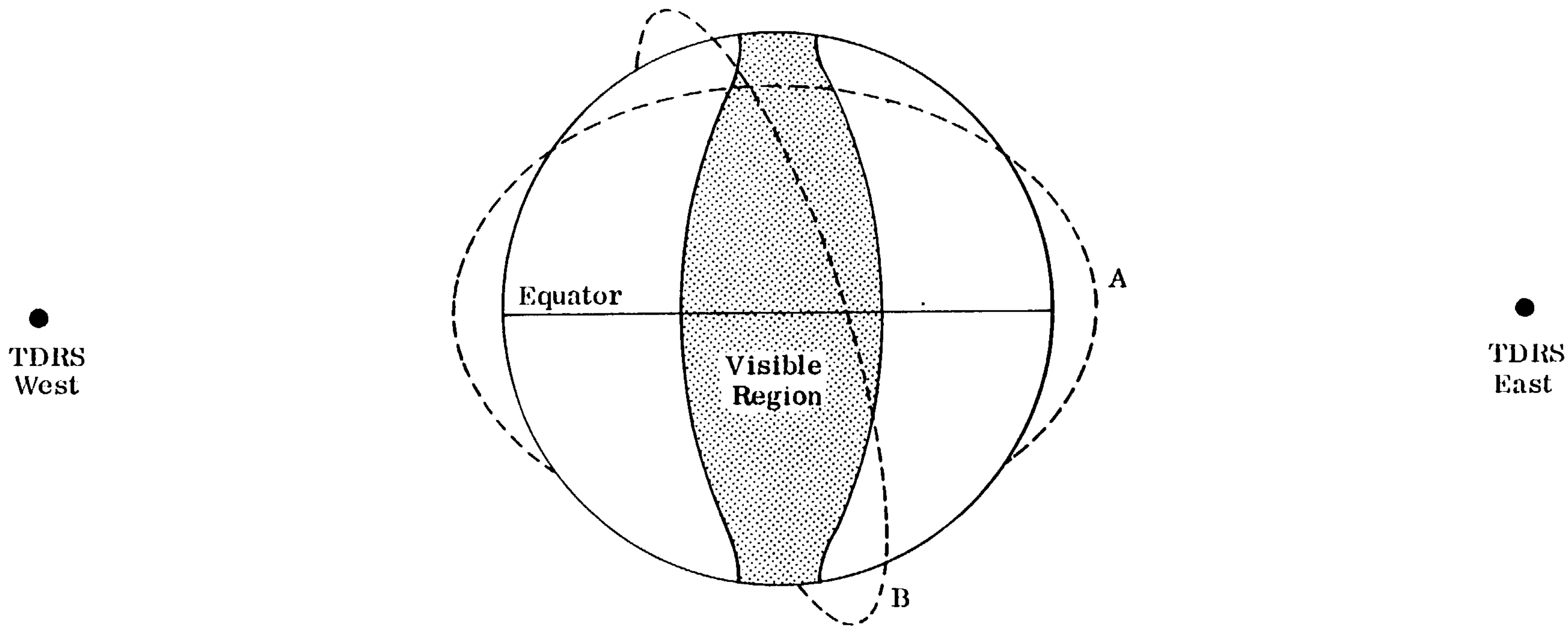
in the Doppler difference measurement. Hence, the partial derivative is given by

$$\frac{\partial D}{\partial \dot{f}_0} = \frac{R_W - R_E}{c}$$

For an equatorial target orbit at an altitude of 1000 kilometers, the value of $R_W - R_E$ reaches a maximum of approximately 7500 kilometers, causing a difference in transmission times of 0.025 second. If the drift rate, $\dot{f}_0$, is equal to 10 hertz per second, this produces a bias in the peripheral measurement of 0.25 hertz. In order to compute this partial derivative, the range values R_E and R_W are needed. Both range and Doppler measurements are simulated by NAP; the range measurement is used by DOPDIF to compute this partial derivative and is then discarded.

2.3 GEOMETRY OF DOPPLER DIFFERENCE TRACKING

Figure 2-1 indicates the geometry applicable in the case of a high-inclination target orbit. The shaded area represents the region within which a low-orbiting user spacecraft is visible from both TDRS relay satellites, permitting Doppler difference data to be taken. The size and shape of this area depend upon the altitude of the user spacecraft, since for higher orbits the visible regions above the poles widen considerably. (For user altitudes greater than 1600 kilometers, there is also a large visible region behind the Earth; however, such altitudes are beyond the scope of this study.) The dashed lines in Figure 2-1 which are marked A and B represent the trajectories of a high-inclination user spacecraft on two passes separated by an interval of approximately 6 hours. It is apparent that a high-inclination satellite will spend considerably more time in the visible region on trajectory A than on trajectory B. The periods during which Doppler difference data are available therefore oscillate between long and short passes, with periods of 12 hours. Equatorial or low-inclination user orbits have short, uniform pass lengths; polar orbits have pairs of very long passes separated by intervals of up to 9 hours, during which a high-altitude user is visible only in the polar regions and a low altitude user is never visible to both relays simultaneously.



NOTE: The shaded area represents the region visible from both relay satellites. In an interval of 6 hours, a high-inclination user satellite will change from short passes through the visible region (Trajectory A) to long passes (Trajectory B).

Figure 2-1. Geometry Applicable to Doppler Difference Tracking

SECTION 3 - ANALYSES AND RESULTS

3.1 TRACKING OF THE RELAY SATELLITES

In order to obtain estimates of the accuracy with which the TDRSS relay satellites will be tracked, one error analysis run was made for each of the two operational relays. The Navigation Analysis Program (NAP) and its associated covariance analyzer (NAPCOV) were used to perform the analysis. TDRS West was tracked from White Sands, New Mexico, via ground transponders at White Sands and at Madrid, Spain. TDRS East was also tracked from White Sands, but via ground transponders at White Sands and Orroral Valley, Canberra, Australia. In each case, the satellite was tracked for 24 hours, taking a 10-minute span of data every 2 hours. Each 10-minute span comprised 5 minutes of tracking via the White Sands transponder and 5 minutes via the remote ground transponder; there was a 10-second interval between adjacent data points. Both range and Doppler measurements were used.

The ground coordinates, data weights, and consider parameter uncertainties used are listed in Table 3-1. Because of a program problem, the ground transponder geodetics could not be included as consider parameters; since the errors arising from these parameters will be comparable to those arising from the tracking station coordinates, which gave very small contributions to the overall uncertainty, this omission is not considered serious.

The results of these analyses are summarized in Table 3-2. In each case, the errors listed are the maximum values attained when the covariances were propagated through the 24-hour data span. It is apparent that although the radial and along-track errors on the two relays are comparable, the cross-track uncertainty is almost an order of magnitude smaller on TDRS West than on TDRS East. This reflects the fact that the western relay is tracked via ground transponders on both sides of the equator, while the transponders for TDRS East are both in the northern hemisphere.

Table 3-1. Data and Error Models for the Relay Satellite Bilateration Tracking

(a) GROUND COORDINATES

Installation	Latitude			Longitude			Height
	Deg	Min	Sec	Deg	Min	Sec	(meters)
White Sands*	32	28	28.91	253	37	48.44	1189.1
Madrid	40	27	16.52	355	49	43.30	793.2
Orroral Valley	-35	37	52.85	148	57	20.91	946.0

*The same coordinates were used for both the tracking station and the ground transponder at White Sands.

(b) DATA

Weights: Range--4.5 meters (one-way)
Doppler--0.003 hertz (one-way)

Data Rate: 6 per minute for each measurement type

(c) CONSIDER PARAMETERS

Station Offset: ±10 meters each direction
Range Bias: ±4.5 meters
Solar Reflectivity: ±0.14 (assuming area/mass ratio = 0.02 m^2/kg)
GM of Earth: ±0.0001%
C(2, 0) ±0.0007%
C(3, 0) ±0.44%
C(4, 0) ±1.8%

(d) RELAY POSITIONS

TDRS East: 303.5 degrees east longitude; 2 degrees inclination
TDRS West: 183.5 degrees east longitude; 2 degrees inclination

Table 3-2. Tracking of TDRS Relay Satellites

Satellite	Component	Maximum Errors From Data Noise	Maximum Errors From Consider Parameters	Significant Consider Parameters
TDRS East (via White Sands and Madrid)	H (meters)	0.7	18	Solar pressure
	L (meters)	2.2	83	GM, solar pressure
	C (meters)	8.0	117	GM, solar pressure
	$\dot{H}$ (mm/sec)	0.05	1.1	Solar pressure, GM
	$\dot{L}$ (mm/sec)	0.11	3.4	Solar pressure, GM
	$\dot{C}$ (mm/sec)	0.58	8.4	GM, solar pressure
TDRS West (via White Sands and Orroral Valley)	H (meters)	0.2	12	Solar pressure
	L (meters)	2.1	90	GM
	C (meters)	2.1	16	GM, solar pressure
	$\dot{H}$ (mm/sec)	0.01	0.8	Solar pressure
	$\dot{L}$ (mm/sec)	0.02	1.8	GM, solar pressure
	$\dot{C}$ (mm/sec)	0.15	1.1	GM, solar pressure

NOTE: Errors are maximum values attained in the 24-hour data span.

3.2 RELAY SATELLITE UNCERTAINTIES IN SST ERROR ANALYSES

In order to perform an error analysis of SST data in which the target or user spacecraft state vector is solved for and uncertainties in the relay state vector are considered, it is customary to specify an a priori covariance in the relay satellite state vector at epoch. Since the NAPCOV Program (the error analysis program used for these studies) assumes uncorrelated consider parameters, this requires that the a priori covariance of the relay state vector be diagonal. However, if a diagonal relay satellite state vector covariance is propagated through several hours, it undergoes considerable secular growth; consequently, if values from Table 3-2 are entered on the diagonal of the epoch state covariance, the uncertainties being considered at a data point some hours away from epoch are much larger than the values entered, which were chosen to represent the maximum uncertainty in the relay state vector at any point in the relay orbit.

One solution to this problem is to specify the full covariance matrix of the relay satellite state vector at epoch, including the nonzero correlations between the position and velocity components. This covariance will then propagate exactly as it did in the error analysis with which it was computed, providing the actual state vector uncertainty at every point in the data span.

When the error analysis software does not permit a priori correlations between the consider parameters, an alternative approach is to enter reduced values in the diagonal relay state vector covariance such that when propagated through the data span the uncertainties remain within the maximum values obtained from the relay tracking error analysis. This is the method used for the present study.

When a relay satellite state vector covariance was specified as diagonal in the HLC coordinate system, it was found that the components at epoch which produced the secular growth pattern were the radial position and along-track velocity. The other four components propagate into oscillatory patterns with stable amplitudes in all six components. The radial position and along-track velocity uncertainties at epoch were therefore set to zero and the magnitudes of the other components were adjusted to provide propagated uncertainties whose maxima approximated the values in Table 3-2. For simplicity, the same uncertainties were used on both relay satellites; the values at epoch and the maximum propagated values are listed in HLC coordinates in Table 3-3. It follows that although the consider parameters are nominally the relay state vector at epoch, association of a solve-for uncertainty with any particular component of a relay uncertainty at epoch will have little significance. The errors arising from each component of a given relay state vector are therefore combined into a single parameter for the purpose of tabulating an error budget.

3.3 DOPPLER DIFFERENCE ANALYSES PERFORMED

Error analysis runs were made using Doppler difference data for a number of target altitudes and inclinations and for a variety of tracking schedules. The target orbits used were approximately circular at altitudes of 300 kilometers and 1000 kilometers and with inclinations of 0, 30, 60, 90, and 120 degrees. The tracking schedules analyzed were the following:

1. From one to four consecutive passes, using all available data with 10-second intervals between data points
2. Three and four consecutive passes, using 5 minutes of data from the center of each pass, with 10-second intervals between data points
3. Same configuration as in 2 above, but with 1-minute intervals between data points

An exception to these configurations was necessary in the case of the target with a 300-kilometer altitude and a 90-degree inclination, which was not visible for more than two consecutive passes. The data weight and consider parameter uncertainties used are given in Table 3-3.

Figure 3-1 shows the geometry of the satellite-to-satellite signal path. The target spacecraft was considered visible from a given TDRS providing that either: (1) the angle α in Figure 3-1 was greater than 90 degrees or (2) the height h of

Table 3-3. Data and Error Models for the Doppler Difference Tracking

(a) DATA

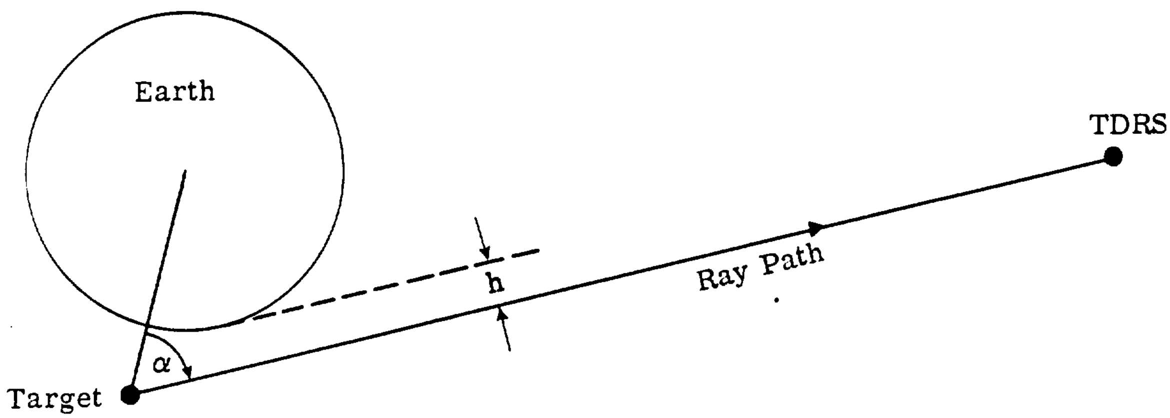
Doppler difference measurements only, via TDRS East and TDRS West
Weight: 0.014 Hz
Data Rate: Six points per minute or one point per minute

(b) CONSIDER PARAMETERS

Spacecraft	Consider Parameter	Value at Epoch	Maximum Value in 6 Hours*
TDRS East	H (m)	0.	20.
	L (m)	15.	70.
	C (m)	100.	140.
	$\dot{H}$ (mm/sec)	1.	3.7
	$\dot{L}$ (mm/sec)	0.	1.5
	$\dot{C}$ (mm/sec)	10.	10.
	Solar Reflectivity	0.14**	
TDRS West	H (m)	0.	20.
	L (m)	15.	70.
	C (m)	100.	140.
	$\dot{H}$ (mm/sec)	1.	3.7
	$\dot{L}$ (mm/sec)	0.	1.5
	$\dot{C}$ (mm/sec)	10.	10.
	Solar Reflectivity	0.14**	
-	Oscillator Bias (KHz)		1
	Oscillator Drift Rate (Hz/sec)		10
	Station Geodetics (m)		10 (each direction)
	Drag (%) (not applied to 1000-km altitude target)		10†
	Geopotentials:		
	GM (%)		0.0001
	C(2, 0) (%)		0.0007
	C(3, 0) (%)		0.44
	C(4, 0) (%)		1.8

*See Section 3.2 for a discussion of the relay satellite state vector uncertainties.
**For area/mass = 0.02 m^2/kg
†Where $C_D A/M = 10^{-3}$ m^2/kg

(c) TDRS LOCATIONS: TDRS East--303.5° east longitude; 2° inclination
TDRS West--183.5° east longitude; 2° inclination



NOTE: Diagram is drawn in the plane defined by the two satellite positions and the center of the Earth.

Figure 3-1. SST Signal Geometry

the ray path above the Earth's surface was greater than 400 kilometers. The second criterion was never met for the 300 kilometer altitude target orbits but afforded considerable additional visibility for targets at 1000 kilometers. For a Doppler-difference data point to be used, the target was required to be visible from both TDRS's.

For each orbital and tracking configuration, the computed tracking errors were rotated into radial, along-track, and cross-track (HLC) coordinates and propagated through the data span and 12 hours beyond the center of the data span. The maximum values attained by the propagated errors were then tabulated in order to provide an estimate of the errors that might be expected in the production of a 24-hour target ephemeris using the Doppler difference tracking sequence centered in the 24-hour time period to be covered.

3.4 RESULTS OF THE DOPPLER DIFFERENCE ANALYSIS

The results of the Doppler difference error analyses are summarized in Table 3-4. For each orbital configuration and tracking sequence, this table shows the maximum root-sum-square (rss) position error attained during the propagation span, the dominant error source, and the direction (radial, H; along-track, L; or cross-track, C) in which the error is most pronounced. More detailed listings of the data configurations encountered and the computed errors are provided in Tables 3-5 through 3-14 given at the end of this section. In addition, the rss position errors in Table 3-4 are plotted as a function of target inclination in Figures 3-2 and 3-3, which are also given at the end of this section.

In order to provide indications of the values assumed and the patterns traced out by the Doppler difference measurement, Tables 3-5 through 3-14 list (to the nearest 10-second increment) the start and end times of each of the four passes of data used in the analysis. Although these times are listed as times of day, their only significance lies in the intervals between them. An intermediate time

Table 3-4. Doppler Difference Summary

Inclination of Target Orbit	Altitude of Target (km)	Run Description / Max. RSS Error & Major Contributors	Four Complete Passes at Six per Minute	Three Complete Passes at Six per Minute	Two Complete Passes at Six per Minute	One Complete Pass at Six per Minute	Four 5-Minute Passes at Six per Minute	Four 5-Minute Passes at One per Minute	Three 5-Minute Passes at Six per Minute	Three 5-Minute Passes at One per Minute
0°	300	RSS Error (m)	371	436	530	5.51×10^{6}	384	420	590	694
		Contributors	Drag, Drift (L, C)	Drag, Drift (L, C)	Drag, Drift (C, L)	Noise (C)	Drag (L, C)	Drag (L, C)	Noise, Drag (C, L)	Noise, Drag (C, L)
0°	1000	RSS Error (m)	251	325	476	6.5×10^{4}	115	154	549	880
		Contributors	Drift, Bias (C)	Drift, Bias (C)	Drift, Bias (C)	Noise (C)	Drift (C, L)	Drift (C)	Noise (C)	Noise (C)
30°	300	RSS Error (m)	314	332	325	1.47×10^{5}	309	313	384	390
		Contributors	Drag (L)	Drag (L)	Drag (L)	Noise (L, C)	Drag (L)	Drag (L)	Drag (L)	Drag (L)
30°	1000	RSS Error (m)	52	58	65	2265	112	121	138	197
		Contributors	Drift (L)	Drift (L)	Drift (L, C)	Noise (L, C)	Drift (L)	Drift (L)	Drift (L)	Drift (L, C)
60°	300	RSS Error (m)	320	316	307	6.0×10^{4}	291	299	350	359
		Contributors	Drag (L)	Drag (L)	Drag (L)	Noise (L)	Drag (L)	Drag (L)	Drag (L)	Drag (L)
60°	1000	RSS Error (m)	55	71	117	788	90	96	77	90
		Contributors	Drift (L)	Drift (L)	Drift (L)	Drift, Noise (L)	Drift (L)	Drift (L)	Drift (L)	Drift (L)
90°	300	RSS Error (m)			1020	2.69×10^{5}				
		Contributors			Drift, Drag, Relays (L)	Noise (L)				
90°	1000	RSS Error (m)	49	58	245	2.1×10^{4}	45	46	75	116
		Contributors	Drift (L)	Drift (L)	Drift (L)	Drift (L)	Drift (L)	Drift (L)	Noise (L)	Noise (L)
120°	300	RSS Error (m)	357	345	338	1.07×10^{5}	359	351	357	356
		Contributors	Drag (L)	Drag (L)	Drag (L)	Noise (L)	Drag (L)	Drag (L)	Drag (L)	Drag (L)
120°	1000	RSS Error (m)	44	58	73	1596	84	83	98	124
		Contributors	Drift (L)	Drift (L)	Drift (L)	Noise (L)	Drift (L)	Drift, Noise (L)	Drift (L)	Noise, Drift (L)

is also included in each pass; this intermediate time corresponds to one of the following:

1. The time at which the maximum measurement value occurs (inclinations of 0, 30, and 60 degrees)
2. The time at which the measurement value changes sign (inclination of 90 degrees)
3. The time at which the minimum measurement value occurs (inclination of 120 degrees)

Also included are the measurement value corresponding to each time and the maximum absolute rate of change undergone by the measurement during the pass. It should be noted that the 5-minute spans of data used in some of the tracking schedules were chosen so as to include the intermediate point listed in the data configuration.

The result portion of each table shows, for each tracking schedule simulated, the number of data points used, the approximate time interval over which data were taken, and the maximum position errors computed for the target in the 12-hour error propagation span. The total rss position error is shown and is also broken down according to direction and source (noise or consider parameters). The directional errors are expressed as radial, along-track, and cross-track components (HLC). The major contributing consider parameters in each direction are also indicated.

3.5 COMPARISON WITH OTHER DOPPLER TRACKING MODES

As an indication of the effectiveness of the Doppler difference tracking compared with standard two-way and one-way Doppler tracking, one of the tracking schedules was run with all three measurement types. The configuration chosen was 60 degrees inclination and 1000 kilometers altitude, with four simultaneous 5-minute passes from each relay. The results are presented in Table 3-15. In the case of the standard one-way Doppler tracking, only the errors arising from the Doppler bias are listed; all other errors are negligible in comparison. These large bias-induced errors are expressed in meters per hertz or bias uncertainty. Since these bias errors may approach hundreds of hertz, it is evident that the standard one-way Doppler is not a practicable tracking mode. However, in this configuration, the differenced one-way Doppler produces errors comparable to two-way Doppler tracking.

3.6 DISCUSSION OF DOPPLER DIFFERENCE RESULTS

It is apparent from Table 3-4 that a single pass of Doppler difference data is not sufficient for orbit determination in any configuration. Since the very large single-pass errors are dominated by data noise, it is unlikely that such a differential correction would converge unless provided with an extremely good a priori state vector.

For data spans which include two or more passes, errors in the 300-kilometer altitude orbit are, in general, between 300 and 400 meters along-track, and they arise chiefly from uncertainties in the drag coefficient. The equatorial orbit at this altitude is also subject to cross-track errors due to both data noise and a number of consider parameters, particularly the oscillator drift rate. Polar orbits have restricted periods of visibility at low altitudes; such configurations are not recommended for this type of tracking.

Since the drag coefficient does not affect the orbits at altitudes of 1000 kilometers, the errors are, in most cases, smaller than for the corresponding 300-kilometer orbit. Root-sum-square errors vary between 50 and 150 meters along-track, except in the case of the equatorial orbit, where large cross-track errors arising from data noise and oscillator drift rate are dominant. In general, errors in the high-altitude orbits are more sensitive to a reduction in the quantity of data used than are those in the low orbits. This is accounted for by the fact that the major error source for the low orbits is a dynamic parameter (drag), for which most of the effects are accumulated during the 12-hour error propagation, while the higher orbit is more heavily influenced by a parameter (oscillator drift rate) which enters the problem as a perturbation in the actual data.

Table 3-5. Doppler Difference Data Configuration and Tracking Errors for a Target at 300 Kilometers Altitude and 0 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:45:00 8:50:40 8:56:00	93565 101483 94036	45.8
2	10:22:10 10:27:40 10:33:10	93630 101366 93768	45.3
3	11:59:10 12:04:50 12:10:20	93193 101217 93515	46.1
4	13:36:20 13:42:00 13:47:20	93183 101061 93760	45.6

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	269	5	371	H 14 L 357 C 194	Drag Drag Drift rate, drag, bias	1 7 33
Three Complete Passes at Six per Minute	202	3.5	436	H 14 L 339 C 303	Drag Drag Drift rate, bias	2 10 42
Two Complete Passes at Six per Minute	134	1.75	530	H 11 L 339 C 409	Drag Drag Drift rate, bias, GM, relays	5 27 164
One Complete Pass at Six per Minute	67	0.25	5510×10^3	H 31 L 448 C 10550	Drift rate, drag Dirft rate, drag Drift rate	16567 220×10^3 5510×10^3
Four 5-Minute Passes at Six per Minute	124	5	384	H 17 L 373 C 177	Drag Drag Drag	3 14 74
Four 5-Minute Passes at One per Minute	24	5	420	H 17 L 375 C 195	Drag Drag Drag	6 32 163
Three 5-Minute Passes at Six per Minute	93	3.25	591	H 21 L 373 C 386	Drag, drift rate Drag Drag, drift rate, bias	12 66 290
Three 5-Minute Passes at One per Minute	18	3.23	694	H 20 L 370 C 375	Drag, drift rate Drag Drift rate, drag, bias	21 117 493

Table 3-6. Doppler Difference Data Configuration and Tracking Errors for a Target at 300 Kilometers Altitude and 30 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:44:10 8:49:50 8:56:00	83093 90329 81764	44.9
2	10:21:30 10:27:50 10:33:50	79310 88012 80071	44.6
3	11:59:40 12:05:40 12:11:00	82283 90577 84438	44.3
4	13:37:50 13:43:10 13:47:40	89270 95766 90908	40.3

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	276	5	314	H 14 L 313 C 24	Drag, drift rate Drag Relays, drift rate	1 5 2
Three Complete Passes at Six per Minute	216	3.5	332	H 13 L 331 C 20	Drag Drag Relays	2 8 4
Two Complete Passes at Six per Minute	147	2	324	H 11 L 324 C 20	Drag Drag Drift rate, relays	4 12 7
One Complete Pass at Six per Minute	72	0.2	147×10^3	H 18 L 640 C 162	Drift rate, drag Drift rate, drag Drift rate	5×10^3 133×10^3 75×10^3
Four 5-Minute Passes at Six per Minute	124	5	309	H 16 L 309 C 23	Drag, drift rate Drag Relays	3 14 5
Four 5-Minute Passes at One per Minute	24	5	313	H 18 L 312 C 24	Drag, drift rate Drag Relays	6 31 11
Three 5-Minute Passes at Six per Minute	93	3.25	384	H 31 L 377 C 54	Drag, drift rate Drag Drag, drift rate	17 54 34
Three 5-Minute Passes at One per Minute	18	3.25	390	H 28 L 371 C 49	Drag, drift rate Drag Drag, drift rate	30 98 60

Table 3-7. Doppler Difference Data Configuration and Tracking Errors for a Target at 300 Kilometers Altitude and 60 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:41:00 8:46:00 8:56:20	55870 59340 44391	45.3
2	10:18:30 10:29:00 10:37:00	37509 49633 42493	35.8
3	12:01:40 12:10:10 12:13:30	54008 65453 63839	43.2
4	13:43:40 13:46:40 13:48:20	83934 85737 85179	19.6

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	306	5	320	H 14 L 319 C 14	Drag Drag Relays	1 5 1
Three Complete Passes at Six per Minute	277	3.5	316	H 15 L 316 C 14	Drag Drag Relays	1 5 1
Two Complete Passes at Six per Minute	205	2	307	H 10 L 307 C 16	Drag Drag Relays	3 13 2
One Complete Pass at Six per Minute	93	0.25	59501	H 21 L 829 C 57	Drift rate, C(4,0), drag " " C(4,0), dr. rate	1599 59367 7672
Four 5-Minute Passes at Six per Minute	120	5	291	H 16 L 290 C 15	Drift rate, drag Drag Relays	2 16 1
Four 5-Minute Passes at One per Minute	23	5	299	H 18 L 297 C 15	Drift rate, drag Drag, drift rate Relays	5 35 3
Three 5-Minute Passes at Six per Minute	93	3.5	350	H 23 L 348 C 15	Drag Drag Relays	9 42 7
Three 5-Minute Passes at One per Minute	18	3.5	359	H 23 L 352 C 15	Drag, drift rate Drag Relays	17 86 13

Table 3-8. Doppler Difference Data Configuration and Tracking Errors for a Target at 300 Kilometers Altitude and 90 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:46:50 8:54:00 9:05:40	19203 0 -22912	45.1
2	10:08:40 10:32:00 10:42:30	-1665 0 11643	44.7
3			
4			

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute						
Three Complete Passes at Six per Minute						
Two Complete Passes at Six per Minute	318	2	1020	H 44 L 1018 C 18	Dr. rate, relays Dr. rate, drag, relays Dr. rate, relays	2 53 0.5
One Complete Pass at Six per Minute	114	0.33	269×10^3	H 360 L 16360 C 323	Dr. rate, relays Dr. rate, relays Dr. rate, relays	6254 268×10^3 6249
Four 5-Minute Passes at Six per Minute						
Four 5-Minute Passes at One per Minute						
Three 5-Minute Passes at Six per Minute						
Three 5-Minute Passes at One per Minute						

Table 3-9. Doppler Difference Data Configuration and Tracking Errors for a Target at 300 Kilometers Altitude and 120 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:54:00 9:01:40 9:08:20	-50801 -63771 -54618	54.0
2	10:16:30 10:24:50 10:33:00	-45609 -59631 -45942	52.4
3	11:42:30 11:48:40 11:54:40	-62678 -71300 -62750	46.6
4	13:10:20 13:14:30 13:17:00	-84216 -88804 -87135	34.9

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	302	4.5	357	H 15 L 357 C 15	Drag, GM Drag Relays, dr. rate	1 5 1
Three Complete Passes at Six per Minute	261	3	345	H 16 L 344 C 16	Drag Drag Relays, dr. rate	1 6 1
Two Complete Passes at Six per Minute	187	1.5	338	H 13 L 337 C 15	Drag Drag Relays	3 7 1
One Complete Pass at Six per Minute	87	0.25	107×10^3	H 35 L 877 C 155	C(4, 0) C(4, 0), dr. rate C(4, 0)	5405 104×10^3 26×10^3
Four 5-Minute Passes at Six per Minute	124	4.25	359	H 18 L 358 C 13	Drag, GM Drag Relays	3 24 2
Four 5-Minute Passes at One per Minute	23	4.24	351	H 16 L 347 C 13	Drag, GM Drag Relays	7 54 4
Three 5-Minute Passes at Six per Minute	93	2	407	H 23 L 399 C 19	Drag, GM, drift rate Drag Dr. rate, relays	14 78 9
Three 5-Minute Passes at One per Minute	18	2	414	H 22 L 390 C 18	Drag, GM, bias Drag Dr. rate, relays	27 143 16

Table 3-10. Doppler Difference Data Configuration and Tracking Errors for a Target at 1000 Kilometers Altitude and 0 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:21:00 8:35:00 8:48:40	63752 96130 64776	70.1
2	10:15:10 10:29:00 10:42:50	64050 95994 64335	70.8
3	12:09:10 12:23:10 12:37:10	63632 95811 63301	71.0
4	14:03:10 14:17:10 14:31:20	63173 95624 63128	71.0

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)		Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	673	6.25	252	H	10	Dr. rate, bias, GM	0.3
				L	56	Drift rate	1
				C	249	Dr. rate, bias, GM	7
Three Complete Passes at Six per Minute	503	4.25	325	H	10	Dr. rate, GM, bias	0.3
				L	62	Drift rate	2
				C	323	Drift rate	12
Two Complete Passes at Six per Minute	334	2.25	476	H	4	Dr. rate, GM	0.6
				L	73	Drift rate	3
				C	473	Dr. rate, bias	18
One Complete Pass at Six per Minute	167	0.5	64902	H	29	Drift rate	191
				L	372	Drift rate	2064
				C	9891	Drift rate	64137
Four 5-Minute Passes at Six per Minute	124	5.75	115	H	8	Dr. rate, GM, bias	2
				L	72	Drift rate	11
				C	94	Dr. rate, bias	59
Four 5-Minute Passes at One per Minute	24	5.75	154	H	9	Dr. rate, GM, bias	5
				L	79	Drift rate	24
				C	135	Dr. rate, bias	132
Three 5-Minute Passes at Six per Minute	93	3.75	549	H	11	Dr. rate, GM, bias	19
				L	45	Drift rate	94
				C	283	Dr. rate, bias	472
Three 5-Minute Passes at One per Minute	18	3.75	880	H	12	Dr. rate, GM, bias	34
				L	44	Drift rate	169
				C	319	Dr. rate, bias	820

Table 3-11. Doppler Difference Data Configuration and Tracking Errors for a Target at 1000 Kilometers Altitude and 30 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:19:50 8:34:00 8:49:40	56798 86145 50175	68.4
2	10:13:30 10:29:10 10:44:20	48730 83210 51117	65.6
3	12:08:40 12:24:20 12:38:00	50593 86470 59292	69.4
4	14:04:10 14:18:20 14:31:20	60294 92222 66489	69.7

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)		Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	707	6.25	52	H	6	GM, drift rate, bias	0.2
				L	47	Drift rate	1
				C	24	Dr. rate, relays	0.4
Three Complete Passes at Six per Minute	543	4.25	58	H	4	GM, drift rate	0.3
				L	53	Drift rate	1
				C	25	Dr. rate, relays	0.6
Two Complete Passes at Six per Minute	366	2.5	65	H	4	Drift rate, GM	0.5
				L	62	Drift rate	2
				C	24	Dr. rate, relays	1
One Complete Pass at Six per Minute	180	0.5	2265	H	20	Drift rate	72
				L	642	Drift rate	1941
				C	185	Drift rate	1061
Four 5-Minute Passes at Six per Minute	124	5.75	112	H	15	Dr. rate, bias, GM	2
				L	109	Dr. rate, bias	13
				C	25	Relays, dr. rate	4
Four 5-Minute Passes at One per Minute	24	5.75	121	H	16	Dr. rate, bias, GM	5
				L	115	Dr. rate, bias	29
				C	26	Relays, dr. rate	9
Three 5-Minute Passes at Six per Minute	93	4	138	H	15	Dr. rate, bias, GM	23
				L	124	" " "	67
				C	23	Relays	46
Three 5-Minute Passes at One per Minute	18	4	197	H	16	Dr. rate, bias, GM	40
				L	121	Dr. rate, bias	152
				C	26	Relays, dr. rate	85

Table 3-12. Doppler Difference Data Configuration and Tracking Errors for a Target at 1000 Kilometers Altitude and 60 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:15:50 8:29:10 8:55:30	42615 59141 2339	49.2
2	10:06:40 10:30:40 10:50:10	10823 46541 22060	37.1
3	12:04:20 12:29:30 12:40:50	4374 65681 52584	60.0
4	14:07:40 14:21:40 14:31:00	57776 86440 74390	65.4

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	862	6.25	55	H 7 L 53 C 18	GM, drift rate Dr. rate, relays bias Relays	0.2 1 0.1
Three Complete Passes at Six per Minute	721	4.5	71	H 6 L 69 C 19	Drift rate, GM Dr. rate, relays Relays	0.3 1 0.2
Two Complete Passes at Six per Minute	501	2.5	117	H 8 L 116 C 18	Drift rate, GM Drift rate Relays	0.4 3 0.3
One Complete Pass at Six per Minute	239	0.66	788	H 18 L 641 C 21	Drift rate Dr. rate, relays Dr. rate, relays	14 458 84
Four 5-Minute Passes at Six per Minute	124	5.75	90	H 12 L 87 C 16	Dr. rate, bias, GM Dr. rate, bias, relays Relays	2 18 1
Four 5-Minute Passes at One per Minute	24	5.75	96	H 13 L 89 C 16	Dr. rate, bias, GM Drift rate Relays	5 35 3
Three 5-Minute Passes at Six per Minute	93	4	77	H 10 L 71 C 16	Dr. rate, GM, bias Dr. rate, relays, bias Relays	6 26 5
Three 5-Minute Passes at One per Minute	18	4	90	H 11 L 75 C 16	Dr. rate, GM, bias Dr. rate, relays, bias Relays	12 52 11

Table 3-13. Doppler Difference Data Configuration and Tracking Errors for a Target at 1000 Kilometers Altitude and 90 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:10:30 8:11:20 8:40:00 8:59:40 9:14:50	43565 41650 0 -28973 -17540	43.8
2	9:53:20 10:05:10 10:21:00 10:31:40 10:[illegible]9:50	7038 0 -7447 0 19808	96.4
3	11:41:10 11:54:00 12:15:10 12:40:50 12:14:20	-31265 -43512 0 57213 56109	59.7
4	13:28:30 13:37:00 13:44:50	-66304 -75729 -66671	38.9

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)	Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	1266	5.5	49	H 7 L 46 C 16	GM, drift rate Drift rate Relays	0.1 1 0.1
Three Complete Passes at Six per Minute	1167	4.5	58	H 7 L 55 C 16	Drift rate, GM Drift rate Relays	0.1 1 0.1
Two Complete Passes at Six per Minute	787	2.75	245	H 21 L 245 C 16	Dr. rate, relays Dr. rate, bias Relays	0.4 9 0.1
One Complete Pass at Six per Minute	387	1	20562	H 558 L 20550 C 562	Drift rate Drift rate Drift rate	20 761 20
Four 5-Minute Passes at Six per Minute	124	5	45	H 9 L 43 C 15	GM, bias Dr. rate, relays Relays	1 4 1
Four 5-Minute Passes at One per Minute	24	5	46	H 9 L 45 C 15	GM, bias Dr. rate, relays Relays	2 10 2
Three 5-Minute Passes at Six per Minute	93	3.5	75	H 7 L 51 C 16	Drift rate, GM Dr. rate, relays Relays	18 55 2
Three 5-Minute Passes at One per Minute	18	3.5	116	H 7 L 51 C 15	Drift rate, GM Drift rate Relays	36 103 3

Table 3-14. Doppler Difference Data Configuration and Tracking Errors for a Target at 1000 Kilometers Altitude and 120 Degrees Inclination

Data Configuration			
Pass Number	Time (UTC)	Measurement Value (Hz)	Maximum Change Rate (Hz/sec)
1	8:20:20 8:50:00 9:06:00	-3310 -63695 -29334	74.6
2	10:06:00 10:25:30 10:45:50	-10305 -58186 -6345	64.0
3	11:47:30 12:02:10 12:20:00	-39707 -71651 -19259	81.7
4	13:30:00 13:41:20 13:53:00	-66975 -90155 -62239	75.8

Run Description	Total Number of Points	Data Duration (hours)	Maximum RSS Error (meters)	Maximum Consider Error (meters)		Major Consider Parameters	Maximum Noise (meters)
Four Complete Passes at Six per Minute	796	5.5	44	H	7	GM, drift rate	0.2
				L	40	Dr. rate, relays	1
				C	17	Relays, dr. rate	0.2
Three Complete Passes at Six per Minute	656	3.75	58	H	6	Drift rate, GM	0.3
				L	55	Dr. rate, relays	1
				C	17	Relays, dr. rate	0.2
Two Complete Passes at Six per Minute	460	2.25	73	H	8	Drift rate, GM	0.4
				L	72	Dr. rate, bias, relays	2
				C	17	Relays, dr. rate	0.2
One Complete Pass at Six per Minute	221	0.5	1596	H	18	Dr. rate, C(4,0)	80
				L	631	Dr. rate, C(4,0)	1426
				C	67	Dr. rate, C(4,0)	346
Four 5-Minute Passes at Six per Minute	124	5	84	H	14	GM, bias, drift rate	3
				L	81	Dr. rate, GM, bias	19
				C	15	Relays, dr. rate	2
Four 5-Minute Passes at One per Minute	24	5	83	H	13	GM, bias, drift rate	7
				L	74	Drift rate, Gm, bias	38
				C	15	Relays, dr. rate	3
Three 5-Minute Passes at Six per Minute	93	3.5	98	H	15	GM, drift rate, bias	13
				L	83	Dr. rate, bias, GM	50
				C	15	Relays	9
Three 5-Minute Passes at One per Minute	18	3.5	125	H	13	GM, drift rate, bias	26
				L	75	Dr. rate, bias, GM	97
				C	15	Relays	18

NOTE: The discontinuity in some of the curves around 90 degrees inclination reflects the fact that only two consecutive passes may be taken for such a target.

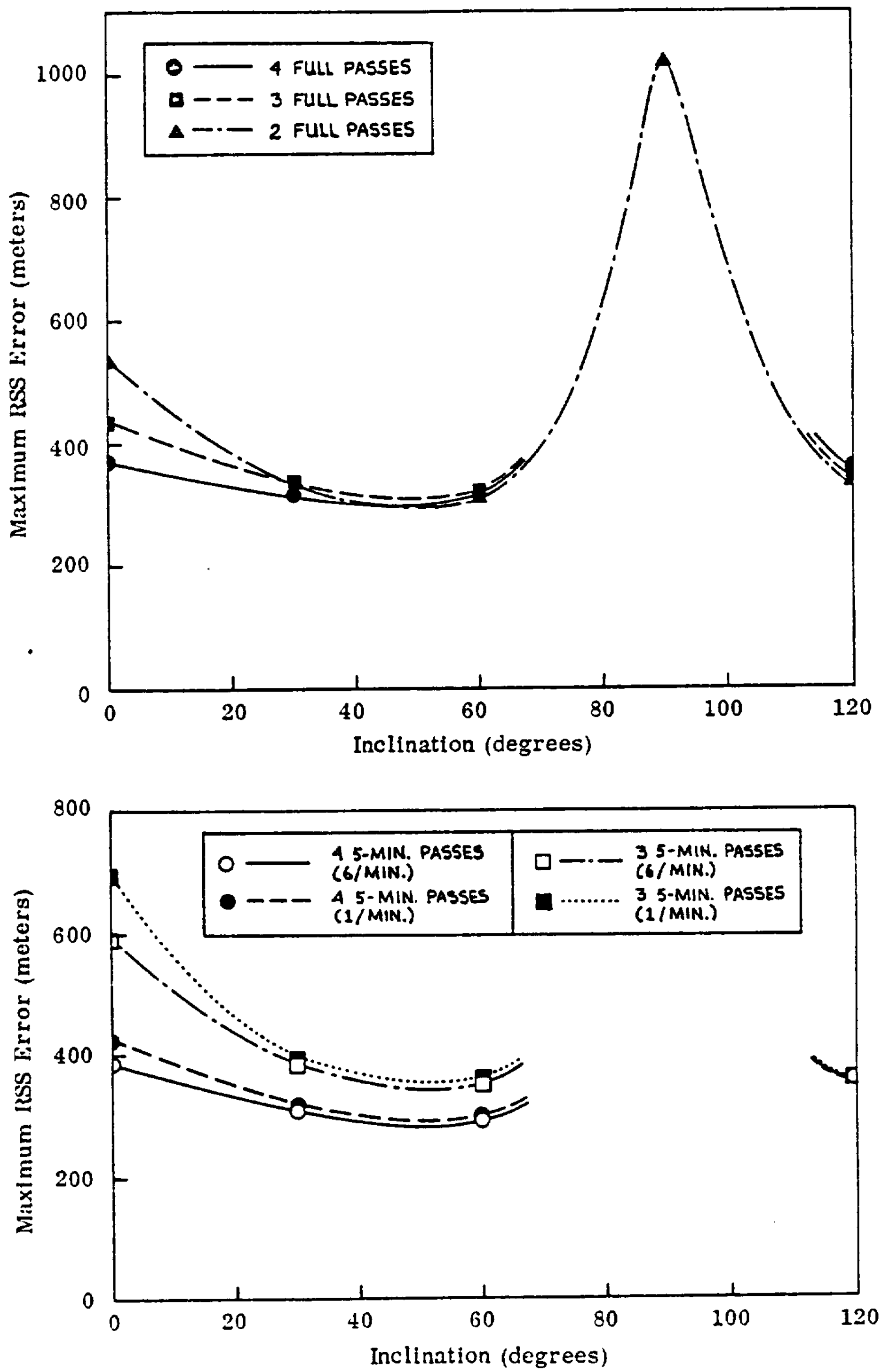


Figure 3-2. Maximum RSS Error for a 300 Kilometer Altitude

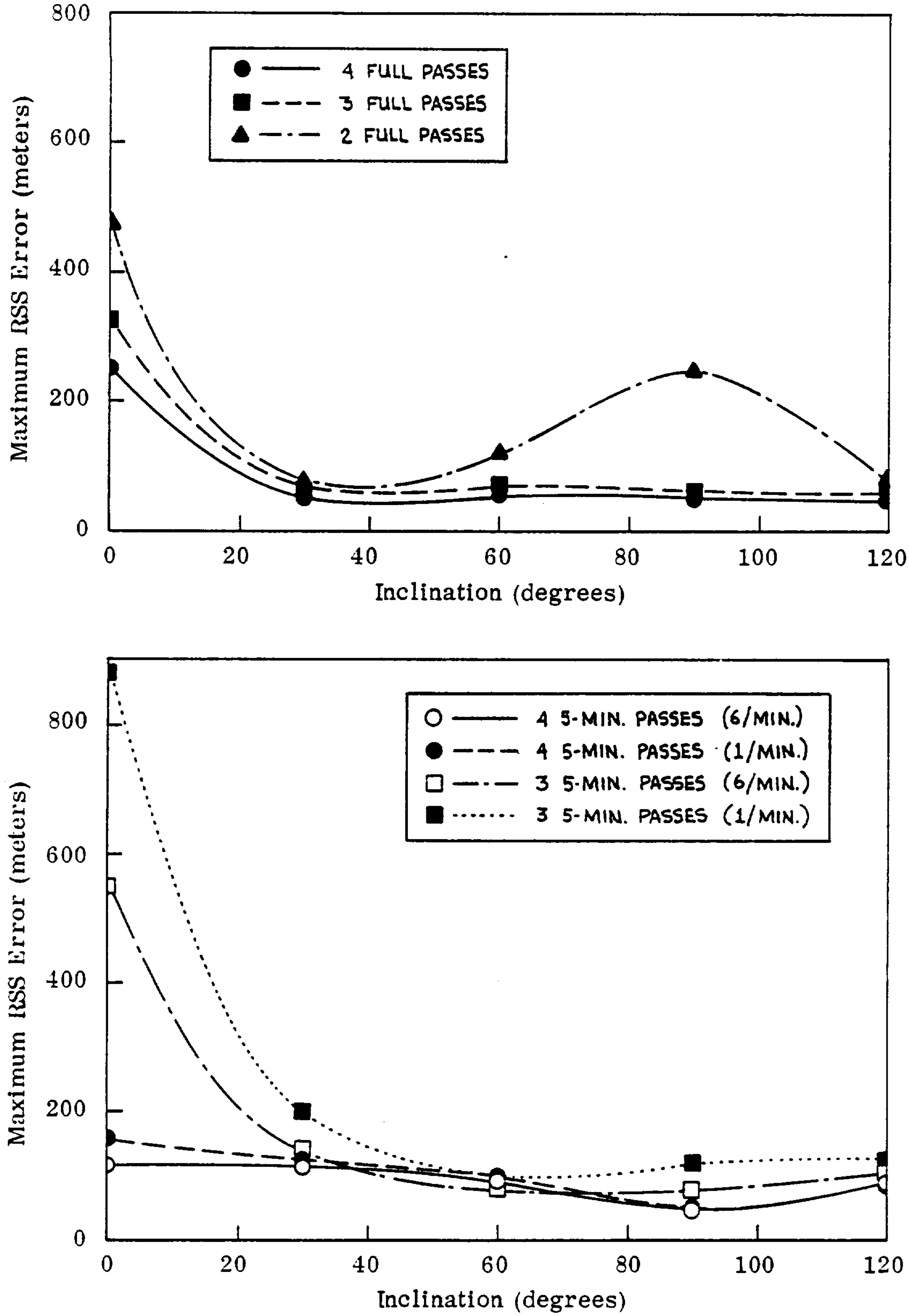


Figure 3-3. Maximum RSS Error for a 1000-Kilometer Altitude

Table 3-15. Spot Comparison Between Doppler Difference and Other Doppler Data Types (60 Degrees Inclination and 1000 Kilometers Altitude)

NOTE: Four 5-minute passes used, with a data rate of six points/minute simultaneously from both relays.

Data Type	Number of Points	Maximum RSS Position Error (meters)	Maximum Consider Errors (meters)	Major Consider Parameters	Maximum Noise Errors (meters)
Two-way Doppler only	124 (from each relay)	43	H 13 L 40 C 18	Relays, GM Relays, GM Relays, GM	1 4 1
Differenced one-way Doppler only	124	90	H 12 L 87 C 16	Drift rate, bias, GM Drift rate, bias, relays Relays	2 18 1
One-way Doppler only	124 (from each relay)	600/Hz	H 92/Hz L 570/Hz C 230/Hz	Bias Bias Bias	1 4 1

SECTION 4 - CONCLUSIONS

This preliminary study of the Doppler difference method of processing one-way transmissions from user spacecraft via TDRSS indicates that it is capable of effectively determining user ephemerides in many configurations. User orbits for which the method is not recommended include low polar or near-polar orbits, which have severly restricted visibility for long periods of time, and equatorial orbits, for which the measurement type is extremely insensitive to cross-track motion. The latter effect may be reduced by relay satellite inclinations greater than the 2 degrees assumed in this study. In addition, it is apparent that the method is not appropriate for processing very short spans of data; at least two passes of the user spacecraft are necessary for an effective state vector determination.

Within these limitations, the Doppler difference measurement has the advantage of providing essentially automatic tracking of users relaying telemetry via the Multiple Access facility without requiring use of signals from the ground. It also eliminates most of the error arising from unknown biases in the user's oscillator frequency. However, its use will impose additional requirements on the user's antenna, which must be capable of transmitting simultaneously to both relay satellites.

REFERENCES

1. Computer Sciences Corporation, CSC/TM-76/6159, Data Processing and Error Analysis for Satellite-to-Satellite Tracking via the Applications Technology Satellite-6 (ATS-6) and the Tracking and Data Relay Satellite System (TDRSS), C. Ayres and G. Rosenblatt, September 1976